Portfolio Theory and Equilibrium

- Simple assumptions:
  - Investors hold well-diversified portfolios ("mean-variance efficient")
  - They all see the same investment opportunity set
  - Short-selling is allowed for risky and riskless assets
    - i.e., borrow money, or borrow stock, sell it, and repurchase it to return it at some future date

Portfolio Theory and Equilibrium

- Results:
  - Any portfolio of efficient portfolios is also an efficient portfolio
  - Since all investors hold efficient portfolios, the sum of their holding must also be an efficient portfolio
  - In equilibrium, supply = demand
    - So the aggregate supply of assets (the "value-weighted market portfolio") must be an efficient portfolio

Capital Market Line

Investment opportunity set from combining portfolios of risky assets with the risk-free asset:

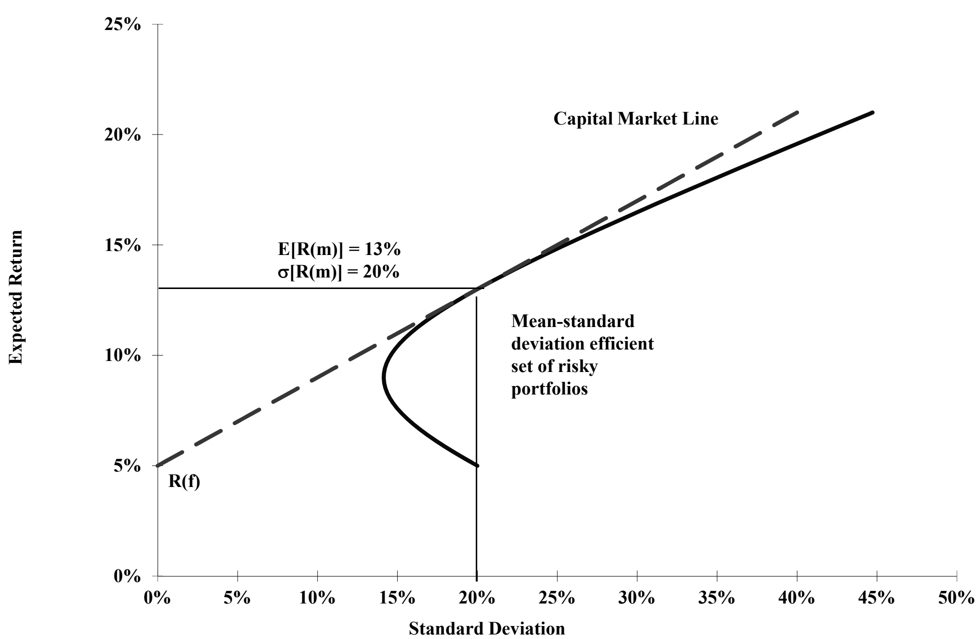
- The tangency portfolio is the only portfolio of risky assets any rational, risk averse investor would hold
- Therefore, it must be the market portfolio of risk assets, R(m)

<u>Equation for the Capital Market Line:</u>

$$E[R(p)] = R(f) + \frac{\{E[R(m)] - R(f)\}}{\sigma[R(m)]} \sigma[R(p)]$$

$$Y = a + b X$$

Capital Market Line



Marginal Risk & Return

Relative to any efficient portfolio, the marginal (incremental) contribution of asset i to the expected return to portfolio p is w(i) E[R(i)], where w(i) is the proportion of asset I in portfolio p and E[R(i)] is the expected return to asset i

Relative to any efficient portfolio, the marginal (incremental) contribution of asset i to the variance of return to portfolio p is w(i) cov[R(i),R(p)], where cov[R(i),R(p)] is the covariance of the return to asset I with the return to portfolio p

Security Market Line

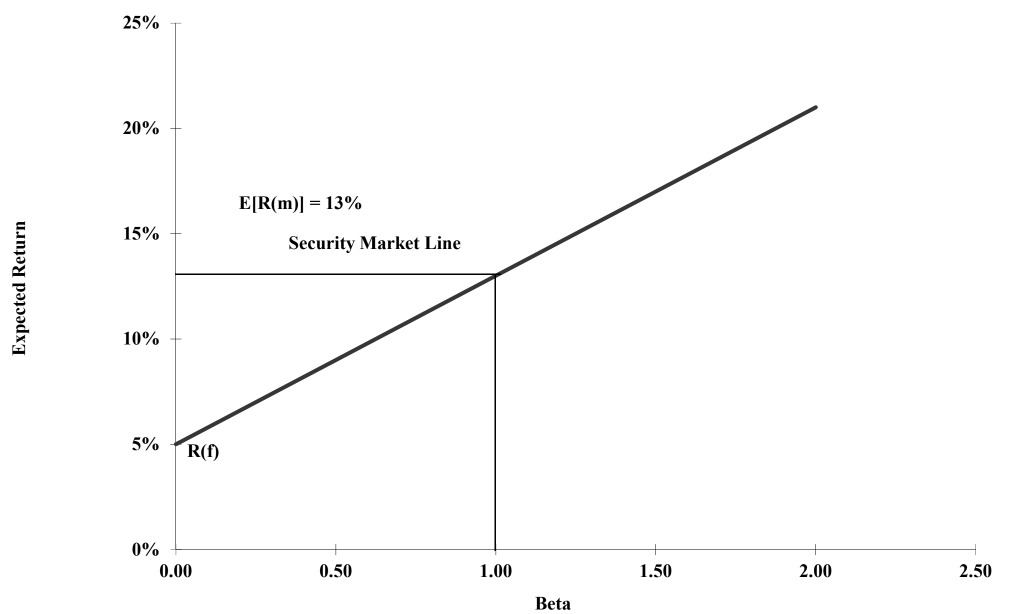
Intuitively, ask how an individual asset contributes to the risk-return trade-off for an efficient portfolio (the Captial Market Line):

$E[R(i)] = R(f) + \underline{\{E[R(m)] - R(f)\}}\ \beta(i)$

where $\beta(i) = cov[R(i),R(m)] / \sigma^2[R(m)]$

Is the relative marginal contribution of asset i to the risk of the market portfolio m

Security Market Line



Sharpe-Lintner CAPM

Basically use the linear equation of the Security Market Line to tell you how much Expected Return should be for any given level of relative non-diversifiable risk (beta)

Impt concept: risk that is diversifiable should not require higher expected returns

In practice, most people now use more general methods of modeling expected returns because of many factors that complicate the use of the simple CAPM

Problems with Using the Sharpe-Lintner CAPM

Finding a good measure of possible investible returns to marketable assets is difficult/impossible:

- most people overload their investment portfolios with "home country" stocks
- most people have non-diversifiable risk in their human capital and their residential real estate
- there may be factors that chance over time and can be hedged by holding particular types of securities, which could create extra demand for those securities that would not be priced by the simple CAPM
- etc.

Empirical Generalizations of the CAPM

- Eugene Fama and Kenneth French have written many papers developing multi-factor CAPMs using multiple portfolios to reflect different risk factors
- e.g., their "three factor" model includes:
  - a "market factor," Mkt-RF
  - a "size factor" SMB ("small minus big" which is the return to a portfolio of stocks with small market capitalization minus the return to high cap stocks), and
  - a "value factor" HML ("high minus low" which is the return to a portfolio of stocks with relatively high book to market value minus the return to low B/M stocks)

Fama-French 3-Factor Model

$[R(it) – R(ft)] = a_i + b_i [R(mt)-R(ft)] + s_i SMB(t) + h_i HML(t) + e(it)$

b_i, s_i, h_i measure the risk exposures of asset i to the three “factors,” market risk, size risk, and value risk

If this model works well to price securities, then the intercept a_i should be small and not reliably different from zero for all assets i

FF3 Market Model Regressions for Swiss Stocks, 2007-2016

Stock	UBS	LOGI	NVS	SYT	ABB	CS	SWZ	EWL
a(i)	-0.0117	-0.0021	0.0023	0.0044	-0.0027	-0.0133	-0.0030	-0.0017
S[a(i)]	0.0079	0.0095	0.0041	0.0061	0.0051	0.0069	0.0032	0.0027
b(i)	1.5098	1.5129	0.6383	0.9402	1.5271	1.3755	0.8626	0.9672
S[b(i)]	0.1930	0.2317	0.1016	0.1485	0.1241	0.1688	0.0781	0.0663
s(i)	-0.4644	0.1985	-0.4975	-0.2991	-0.3997	-0.6421	-0.3157	-0.4039
S[s(i)]	0.3694	0.4435	0.1944	0.2842	0.2376	0.3231	0.1494	0.1269
h(i)	0.8223	0.0353	-0.0448	-0.8546	-0.3316	0.6464	-0.0131	0.0354
S[h(i)]	0.3000	0.3602	0.1579	0.2309	0.1930	0.2624	0.1214	0.1031
Rsq	0.4477	0.3289	0.2662	0.2769	0.5844	0.4487	0.5418	0.6758
S[e]	0.0852	0.1023	0.0448	0.0656	0.0548	0.0745	0.0345	0.0293

FF3 Market Model Regressions for Swiss Stocks, 2007-2016

- None of the intercept estimates is more than 2 std errors from 0
- “Beta” estimates are similar to single factor market models from before
- Small firm factor coefficients are significantly negative for Credit Suisse and Novartis (not surprising since these are huge firms . . .)
- This is also true for the Swiss portfolios SWZ and EWL, since those value-weighted portfolios look like “large cap” stocks
- Value factor coefficients are significantly positive for UBS and Credit Suisse (which have a lot of “assets in place”), and significantly negative for Syngenta (a “growth” firm)

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