Autocorrelated Heteroskedasticity

• Suppose you have regression residuals
  • Mean = 0, not autocorrelated

• Then, look at autocorrelations of the absolute values of the residuals (or the squares of the residuals)

• This tells you if there is heteroskedasticity that varies over time
Example: Xerox Stock Returns

Kurtosis and wide, then narrow, bands in plot are hints of conditional heteroskedasticity

Example: Xerox Stock Returns

After estimating a regression with just a constant for the Xerox returns, the squared residuals have small positive autocorrelations
GARCH Model

Default model is GARCH(1,1), which is not a bad starting point.

Conditional sd graph shows brief periods of high volatility.
GARCH(1,1) Model

\[
R_t = \mu + \varepsilon_t \\
\varepsilon_t \sim N(0, \sigma_t^2) \\
\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\]

Where
\[
\begin{align*}
\mu & \text{ is the mean of the returns} \\
\sigma_t^2 & \text{ is the variance of the errors at time } t \\
\varepsilon_{t-1}^2 & \text{ is the squared error at time } t-1 \\
\omega / (1 - \beta_1 - \alpha_1) & \text{ is the unconditional variance} \\
\alpha_1 & \text{ is the first (lag 1) ARCH parameter} \\
\beta_1 & \text{ is the first (lag 1) GARCH parameter}
\end{align*}
\]

This looks a lot like an ARMA(1,1) model for the squared errors (as deviations from their forecasts),
\[
\nu_t = (\varepsilon_t^2 - \sigma_t^2) \\
\varepsilon_t^2 = \omega + (\alpha_1 + \beta_1) \varepsilon_{t-1}^2 + \nu_t - \beta_1 \nu_{t-1}
\]

Often the GARCH parameter \( \beta_1 \) is close to 1, implying that the movements of the conditional variance away from its long-run mean last a long time

For Xerox \( \beta_1 = .76 \) and \( \alpha_1 = .18 \), so the implied AR(1) parameter is about .94 and the MA(1) coefficient is .76
GARCH Model Diagnostics

In Eviews, most of the residual diagnostics for GARCH models are in terms of the standardized residuals [which should be N(0,1)]

Note that kurtosis is smaller (still not 3, though)

The correlogram for the standardized squared residuals now looks better
EGARCH(1,1) Model

This model basically models the log of the variance (or standard deviation) as a function of the lagged log(variance/std dev) and the lagged absolute error from the regression model.

It also allows the response to the lagged error to be asymmetric, so that positive regression residuals can have a different effect on variance than an equivalent negative residual.

EGARCH(1,1) Model

“GARCH” is the variance the residuals at time $t$

The persistence parameter, $c(5)$, is very large, implying that the variance moves slowly through time.

The asymmetry coefficient, $c(4)$, is negative, implying that the variance goes up more after negative residuals (stock returns) than after positive residuals (returns).
EGARCH Model Diagnostics

The correlogram for the standardized squared residuals still looks pretty good.

In Eviews, most of the residual diagnostics for GARCH models are in terms of the standardized residuals [which should be N(0,1)]

Note that kurtosis is smaller (still not 3, though)
EGARCH Model Extensions

Plotting the log of Xerox’s stock price on the right axis, versus the two estimates of the conditional standard deviation [from GARCH(1,1) and EGARCH(1,1)], you can see that the crash in the stock price occurs at the same time as the spike in volatility, and volatility declined as the stock price slowly recovered.

EGARCH Model Extensions

Include the lagged log of Xerox’s stock price as an additional variable in the EGARCH equation, but it doesn’t add much.
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